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# Deflagration fronts and compressibility

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This paper seeks to bring together a number of results pertaining to the interaction of pressure waves with premixed combustion fronts. Some of the earlier results concerning low-speed flames and acoustics are set in context with some of the later work, which is still being developed and which takes compressibility into account to a much greater extent. The interaction of low-speed flames with steep pressure drops shows a possible mechanism for flame extinction, and the interaction with steep pressure rises indicates conditions for rapid flame acceleration to fast convection–reaction driven deflagrations. This work demonstrates that the transient nature of fast flames, with their corresponding acoustic and reactive acoustic zones due to temperature singularities (blow up) occurring near the driving piston of such deflagrations, is very sensitive to imposed pressure disturbances. In a separate section at the end of this paper, we address the question of the amplification of long-wavelength acoustic waves reflected from fast deflagrations, where the entropy change across such fronts is significant, but where the structure of the deflagration is uncertain.

**Keywords: deflagrations; compressibility; pressure waves; acoustics; resonance; flame extinction**

## 1. Introduction

Acoustic interactions are known to be crucial in the development of flames with possible quenching or acceleration. In an earlier review (McIntosh 1995*a*), the main emphasis concerned two-dimensional effects due to the Rayleigh–Taylor instability driven by the pressure gradient. In this present work the focus is on compressibility and one-dimensional interactions, with particular interest in the different length- and time-scales in such problems. There is a surprising wealth of possible interactions, even in one dimension. To a great extent this work complements that of Clarke, Dold, Kapila and other authors (e.g. Clarke & Cant 1984; Kassoy & Clarke 1985; Singh & Clarke 1992; Dold *et al*. 1991; Short & Dold 1993), who have considered the initial formation of combustion waves from the interaction of a piston with a combustible and initially quiescent gas. The work of the author has been to address the question, 'How do *existing* combustion fronts interact with pressure disturbances?' For low-speed flames, there is an important set of natural time- and length-scales that leads to instructive formulae connecting the pressure disturbance at the flame to the pressure waves on either side. Such an analysis shows clearly that the mass burning rate is very sensitive to pressure changes, and that equally important to the magnitude of the pressure change is the *rate* at which such changes are made. A natural follow-on to those investigations was to consider the effect of sharp rarefactions and shocks on existing low-speed flames. The analysis of both types of interaction was developed using theoretical and numerical approaches. The first type of interaction (rarefactions) showed the emergence of important criteria for flame extinction that are relevant not only to the laminar flames used in the simple analysis, but which also have implications for the laminar flamelet regime of turbulent flames—pressure drops of a sufficient magnitude can cause local extinction. The second type of interaction (shocks) showed the immediate effect of flame 'thinning' (i.e. the reaction zone for a significant time became much narrower and faster)—here was clearly a mechanism for acceleration. Therefore, the next logical question to ask was, 'Could such an acceleration be in some way maintained?' In that high-speed (subsonic) combustion, waves are usually of a transient nature, with diffusion relegated in importance; there is, in fact, no immediately obvious self-sustaining mechanism to replace the classical reactive–diffusion balance which lies at the heart of the mass burning rate eigenvalue for low-speed premixed combustion waves. Thus, in order to practically maintain the acceleration of the deflagration, there needs to be a sustained input—usually a piston maintaining the pressure build up from the rear of the combustion front. The answer then to the question concerning whether the acceleration can be *eventually* self-sustained is undoubtedly that this is possible if there is shock-ignition in *advance* of the combustion front, and this in turn may only be possible given a strong enough initial shock interaction with the low-speed flame. The results of these investigations (where low-speed flames are accelerated by a sharp shock interaction into fast subsonic convective–reactive fronts), are not yet conclusive concerning the emergence of *secondary ignition* (which term is used to describe the formation of reaction centres—shock-induced singularities—ahead of the combustion front). It should always be noted that a fast convective–reactive flame is not a truly stable combustion wave (Booty 1994, 1996; Johnson *et al.* 1996*a*,*b*), since compressible effects in the induction zone ahead of the reaction zone will always lead to thermal runaway taking place near the reaction front. If there are circumstances where it can be shown that the ignition event occurs at the induction zone side of the fast-flame region, then this would herald a different type of behaviour—the onset of separate reaction centres ahead of the first front, and consequently the possibility of rapid self-sustained acceleration of the front, leading to a detonation.

This then leads to a further development of thought. If the front is (by some means) already travelling fast enough to produce significant compressibility, there is then the possibility of the combustion front sending its own compression waves (i.e. strong acoustic disturbances) through the combustible mixture. Thus the last part of this work considers acoustics interacting with a fast (but still subsonic) combustion front. The significant work of Ni & Goel (1995) has led the way in this interesting development. The combustion front, though one dimensional, may in fact be turbulent in its structure, and the unsteady pressure disturbances are now characterized by a long length-scale. Thus shocks interacting with this flame structure are no longer considered in this section of the work. Essentially, one is considering the classical Rankine–Hugoniot analysis of a fast subsonic front, with some *a priori* knowledge of how the mass burning rate of the front changes with pressure.

## 2. Time- and length-scales

Careful analysis has identified key length and time ratios in flame–pressure interactions, defined as follows (see McIntosh 1991, 1993; Batley *et al.* 1993):

$$\tau \equiv \frac{\text{diffusion time}}{\text{acoustic time}}, \qquad N \equiv \frac{\text{characteristic length of pressure disturbance}}{\text{diffusion length}},$$

and which characterize each type of interaction. By defining the Mach number of flame propagation, i.e.

$$M \equiv \frac{u'_{01}}{a'_{01}}$$

(where $u'_{01}$ is the initial burning velocity and $a'_{01}$ the sound speed), it follows that

$$\tau = \frac{1}{NM}.$$

If the flame is characterized by one overall Arrhenius reaction with a non-dimensional activation energy defined as

$$\theta \equiv \frac{E'_{\mathrm{A}}}{R'T'_{\mathrm{b}}}$$

(where $E'_{\mathrm{A}}$ is the dimensional energy, $R'$ is the universal gas constant and $T_{\mathrm{b}}$ is the steady burnt temperature), then the characteristic length-scale of the pressure disturbances determines four distinct cases of pressure–premixed flame interaction.

(i) $N \gg 1/M$: $\tau \ll 1$. Large length-scale disturbances. Pressure gradients *not important throughout* the combustion region (including inner reaction zone *and* outer combustion zones—preheat and equilibrium). The effect of the pressure disturbances are felt only in the outer Eulerian (hydrodynamic) zones, where conservation of momentum and energy implies the acoustic equations for small-amplitude disturbances. For high-speed subsonic fronts, jump conditions emerge from Rankine–Hugoniot jump conditions across the whole combustion region.

(ii) $N = 1/M$: $\tau = 1$. Pressure gradients *not important* in the combustion region; inner reaction zone not affected by pressure field. The effect of the pressure disturbances is predominantly in the outer combustion zones (preheat and equilibrium), where the equations and jump conditions govern the connection between the mass flux and pressure transients.

(iii) $N = 1/\theta^2 M$: $\tau = \theta^2$. Pressure gradients *still not felt* in the combustion region; however, fast time-scale now causes the pressure changes to affect the inner reaction zone. A different equation to that in (i) above now determines the connection between the mass flux and the pressure changes.

(iv) $N = 1$: $\tau = 1/M$. Pressure gradients *now important* in the combustion region, which experiences the full effect of any pressure wave passing through. Non-constant wave speed with nonlinearities for large-amplitude disturbances; the pressure changes are of an ultra-short length-scale.

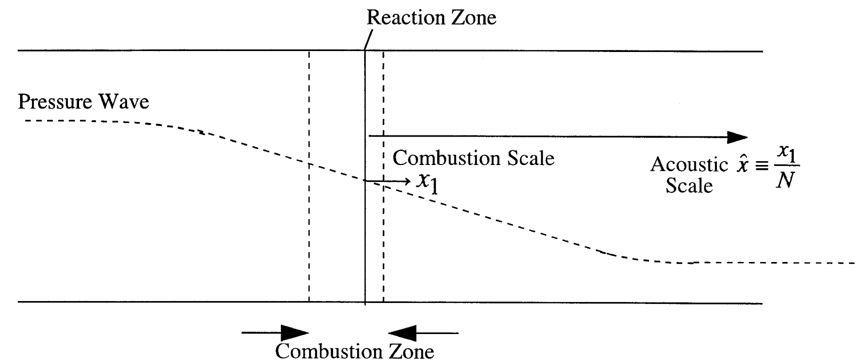


Figure 1. Typical length- and time-scales for pressure interactions with premixed combustion fronts.

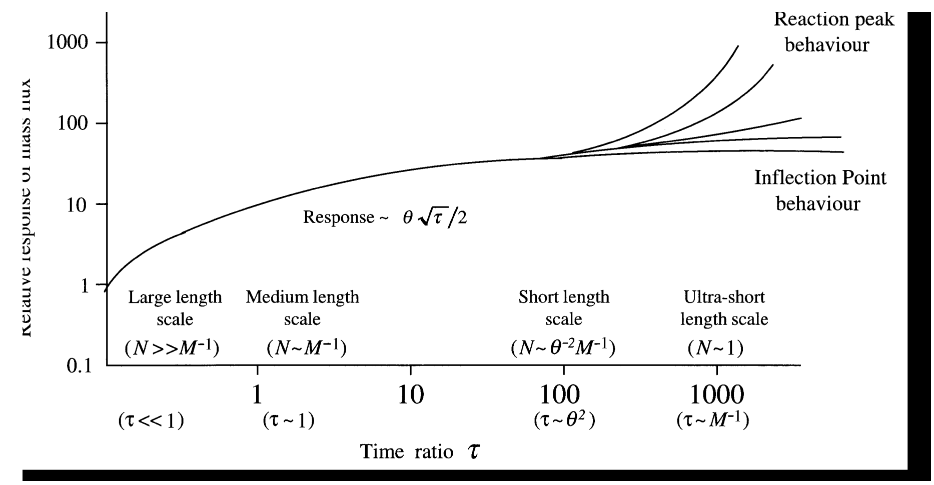


Figure 2. Schematic of mass flux response $((m_0 - 1)/[(1 - \gamma^{-1})(p_0 - 1)])$ to small-amplitude disturbances for different time- and length-scales.

The schematic (figure 1) shows the different order of magnitude of mass flux response, according to the ratio $\tau$ (which is equal to $1/NM$). This diagram pertains to low-speed flames and small-amplitude disturbances. On the left of the diagram, the flame can be regarded as a contact discontinuity where the whole combustion region is swept along with the fluid disturbance. On the right of the diagram, the pressure disturbance is of an ultra-short length-scale such that the pressure gradient is 'felt' within the combustion region. For sharp pressure increases, the (forward) inflection point responds at a faster rate than the reaction peak point, thus 'thinning' the flame, which will have an overall increase in burning velocity before eventually settling down to a new steady-state structure.

## 3. Free flame acoustic resonance

For small-amplitude disturbances, where the initial combustion wave is a low-speed flame, such that pressure changes across the deflagration are negligible, then the most instructive case is case (ii) in the above list. When a premixed flame is near an oscillatory pressure field, it can be shown that there is an important coupling between the strength of the pressure disturbance $\bar{p}_{u0}$ and the fluctuating mass burning rate $\bar{m}_{u0}$ (Ledder & Kapila 1991; McIntosh 1991, 1995*b*), given by

$$\bar{m}_{u0} = \frac{(1-\gamma^{-1})(2r-Q)(\frac{1}{2}+r)\theta\bar{p}_{u0}}{4r}, \tag{3.1}$$

where $r = \sqrt{\omega + \frac{1}{4}}$ and $\omega$ is a non-dimensional frequency, $Q$ is the non-dimensional heat release (typically $Q \approx 0.8$) and $\theta$ is the dimensionless activation energy (typically $\theta \approx 10$).

It should be noted that the result above for laminar flames can equally well be applied to the *thick turbulent flame regime*, where the structure is such that the reaction zone can be thought of as a thick turbulent brush. Using different length-scales determined by viscous diffusion and heat transfer, the basic physics of the interactions described earlier for laminar flames—certainly as regards extinction—can be regarded as similar and thus used to predict the behaviour of thick turbulent flames in a changing pressure environment. For such an application to thick turbulent flames, a first approximation must be that the change in mass burning rate due to the small-scale increase in baroclinically generated vorticity within the flame will not be large. This is an acceptable assumption for case (ii) type interactions, since long-wavelength acoustics are considered and the pressure gradient is not 'felt' in the reaction zone.

Equation (3.1) is for case (ii) ($N = 1/M$: $\tau = 1$). Clearly, as $\omega$ becomes large, then equation (3.1) implies

$$\bar{m}_{u0} \approx \tfrac{1}{2}(1-\gamma^{-1})\theta\bar{p}_{u0}\sqrt{\omega}, \tag{3.2}$$

which, in figure 2, corresponds to the $\frac{1}{2}\theta\sqrt{\tau}$ part of that schematic. In McIntosh (1993), further investigations were undertaken for the case of very high frequency (that is, $\omega \sim O(\theta)$), and, as a result of the asymptotic analysis with this assumption (i.e. case (iii) ($N = 1/\theta^2 M$: $\tau = \theta^2$)), the $\sqrt{\omega}$ 'tail' of equation (3.2) can be shown not to carry on to become larger and larger without limit for increasing frequency. Eventually, the response reaches a peak level, as shown in figure 3.

For $\theta \approx 10$ and $Q = 0.8$ (typical for hydrocarbon combustion), the response peak is thus

$$\left[\frac{m_{u0}}{(1-\gamma^{-1})p_{u0}}\right]\bigg|_{\text{peak}} \approx 40 \tag{3.3}$$

at $\omega_{\text{i max}} \approx 0.75\theta^2 Q^2$, so that there is, in fact, a high-frequency *natural* resonance (that is, apart from any organ pipe resonance from equipment surrounding the flame). In dimensional terms, the resonant frequency is given by

$$f'_{\text{resonance}} \equiv \frac{\omega'_{\text{i resonance}}}{2\pi} \approx \frac{0.75}{2\pi}\left(\frac{E'_{\text{A}}}{R'T'_{\text{b}}}\right)^2 \frac{u'^2_{01}}{\kappa'}\ \text{Hz}. \tag{3.4}$$

This is for an overall one-step reaction, where $u'_{01}$ is the steady burning velocity and $\kappa'$ is the thermal diffusivity. Thus a natural resonant high frequency (of the order of kHz) exists for most practical hydrocarbon flames and should be measurable experimentally, although this has not yet been confirmed. A pure tone generator directed at a laboratory premixed flame should give resonance in the flame itself at

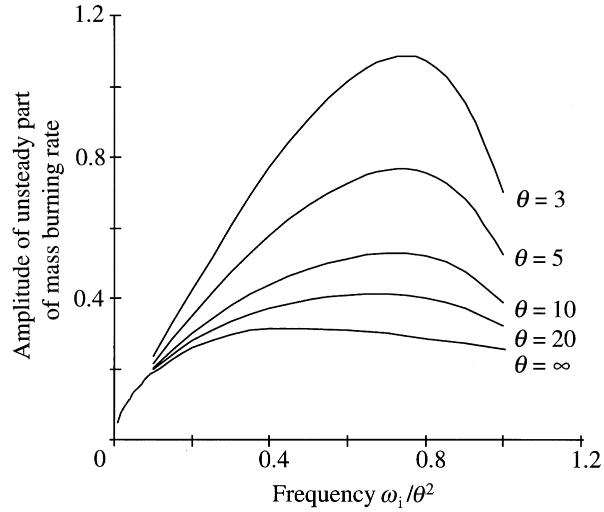


Figure 3. Variation of mass burning ratio $\tilde{M} = \bar{m}_{u0}/[\theta^2 Q(1-\gamma^{-1})\bar{p}_{u0}]$ with frequency and activation energy for high-frequency oscillations, $\tau \sim \theta^2$.

a particular value of high frequency. Clearly, there will be difficulties in measuring a pure harmonic at high frequency, and secondly, the single-step reaction kinetics is an assumption that is difficult to justify—certainly for real hydrocarbon chemistry. However, it is surprising how well the single-step chemistry has worked for flame stability analysis, so, at least qualitatively, equation (3.4) should yield a reasonable approximation to the acoustic resonant frequency for premixed flames. Using a thermal diffusivity for air $\kappa'$ at 1000 K (Incropera & DeWitt 1996 (see p. 839 for an estimate of thermal diffusion coefficient at high temperature)) of $1.69 \times 10^{-4}\ \mathrm{m^2\,s^{-1}}$, a typical burning velocity of $0.2\ \mathrm{m\,s^{-1}}$, with $E'_a/R'T'_b = 10$, yields a typical resonant frequency prediction for hydrocarbon premixed combustion to be *ca.* 2.8 kHz.

## 4. Sharp pressure changes where the structure of the flame is affected

This case is relevant to interactions where $\tau \sim O(M^{-1})$. If the length-scale is small enough, then the pressure gradient is significant in the reaction zone, and there is severe distortion of this region, such that for a pressure drop, the flame broadens and slows down, and for a pressure rise, the reverse happens with the flame thinning and accelerating. The papers by Batley *et al.* (1993) and Johnson & McIntosh (1995) show that sharp pressure changes can cause major changes in the flame structure—transient stretching or compression of the flame—and, consequently, the mass burning rate can alter substantially.

### (*a*) *Extinction caused by steep pressure drops: rarefaction waves*

Undoubtedly in practical situations flame extinction locally will occur due to flame stretch (i.e. two-dimensional) effects. However, extinction can take place even in one dimension. When a sufficiently large pressure drop across a flame causes the flame combustion zone to dilate to such an extent, then the overall mass burning rate drops and does not recover. This is shown schematically in figure 4, which is typical

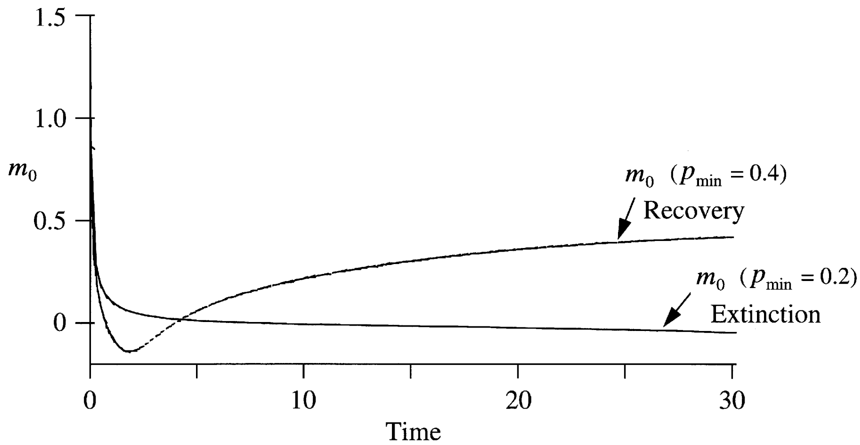


Figure 4. The mass flux response of a premixed flame to a sudden decrease in pressure. Initial pressure $p = 1$, activation energy $\theta = 10$ and heat release $Q = 0.8$. The effect of pressure level is displayed for the two cases of recovery ($p_{\min} = 0.4$) and extinction ($p_{\min} = 0.2$). (After Johnson *et al.* 1995.)

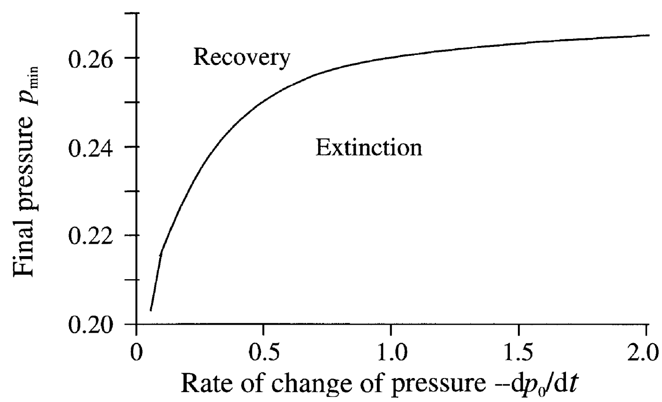


Figure 5. The boundary between extinction and recovery in ($p_{\min}, \mathrm{d}p_0/\mathrm{d}t$) parameter space, for activation energy $\theta = 10$ and heat release $Q = 0.8$. (After Johnson *et al.* 1995.)

of the results for a one-step reaction (see McIntosh 1995*a*; Johnson *et al.* 1995). What becomes very clear is that the rate of pressure drop, as well as the level of final pressure, is very significant (see figure 5).

The possibility of either extinction or flame acceleration due to sharp pressure changes with flames can, in fact, be used in laminar flamelet models of turbulent premixed combustion to predict flamelet behaviour—in particular, *local* extinction events, which may occur in engines, for example.

### (*b*) *Acceleration of low-speed flames*

As the steep pressure rise (which can be a shock) goes through the flame from either direction, the flame thickness contracts greatly as the flame settles to a new (shock-heated) temperature environment, and the mass burning rate increases with consequent acceleration of the flame (see figure 6). This effect has been extensively reported on in the literature (Batley *et al*. 1993). After this initial acceleration, the question which clearly had to be addressed was, 'Could such an acceleration be maintained?' In the *immediate* aftermath of such a *single*-shock interaction, there is no mechanism in the one-dimensional formulation to sustain this acceleration (clearly, there are turbulent flow mechanisms, once two- and three-dimensional effects are allowed, as discussed in Lee (1986)), but these are not the main purpose of the discussion here). The recent work of Dold *et al*. (1995) strongly suggests the possibility of enhanced development of an accelerated combustion front by the accumulation of ignition events from a pressure pulse. That work was with no initial flame present, but suggestive of the mechanism of *forward repeated ignition* as being a primary one-dimensional route for acceleration to detonation. Progress has been made towards investigating this phenomenon by setting up a model problem analytically (Kassoy & Clarke 1985; Blythe & Crighton 1989; Dold *et al*. 1991) and numerically (Singh & Clarke 1992; Chue *et al*. 1993), whereby a moving boundary (piston) on the hot side of the combustion maintains the initial fast convective–reactive driven flame (first proposed by Clarke (1983)). To investigate the subsequent development from a slow flame to a fast convective–reactive flame, one can consider the system of a shock wave passed through a low-speed diffusion–reaction driven flame (Johnson *et al*. 1996*a*), and the corresponding shock strength input to the system of fast-flame, induction zone and shock (see figure 7). Essentially, the investigations centre around the solution of the reactive-Euler equations for small perturbations. This leads to the Clarke equation in the induction zone (Clarke & Cant 1984), which describes the acoustic/explosive events that can occur in the vicinity of a contact surface advancing through a combustible mixture (either a piston or shock wave). As the fast flame evolves with time, as expected, thermal runaway (a temperature singularity) develops on the flame side of the small intermediate zone near the fast flame. Defining the blow-up time as when the reduced temperature,

$$\phi \equiv \frac{E'_{\mathrm{A}}}{R' T_{\mathrm{a}}'^2}(T' - T'_{\mathrm{a}}),$$

reaches the value 5, one can compare the time history of the singularity with that for the case of the piston alone. The ignition event is notably faster with the flame present than without (Johnson *et al*. 1996*a*).

The consequences of this work are that after a shock wave has gone through a flame from the hot side into the unburnt mixture, if the amplitude of the shock is sufficiently strong, then at a comparatively large distance (in diffusion terms) from the flame, the Clarke equation,

$$\left\{\frac{\partial^2(\phi_{\hat{t}})}{\partial \hat{t}^2} - \frac{\partial^2(\phi_{\hat{t}})}{\partial \hat{x}^2}\right\} - \left\{\gamma\frac{\partial^2(\mathrm{e}^{\phi})}{\partial \hat{t}^2} - \frac{\partial^2(\mathrm{e}^{\phi})}{\partial \hat{x}^2}\right\} = 0, \tag{4.1}$$

is invoked (which describes acoustics in an explosive mixture) at the same time as the flame itself is still undergoing transient contractions and dilations as a result of the sharp initial pressure change. For this case, the flame experiences a sharp increase in its burning rate $m_0$ so that under transient conditions one has the quasi-steady 'fast flame' or 'convected explosion', which is then coupled with the shock through an induction zone where acoustics and chemistry combine. The interaction between the induction zone and the fast flame provides the mechanism for acoustic

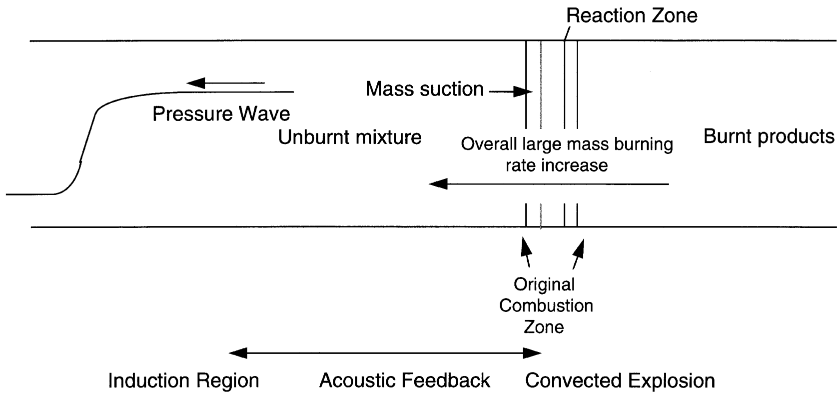


Figure 6. Transient flame contraction and acceleration after a sharp pressure increase.

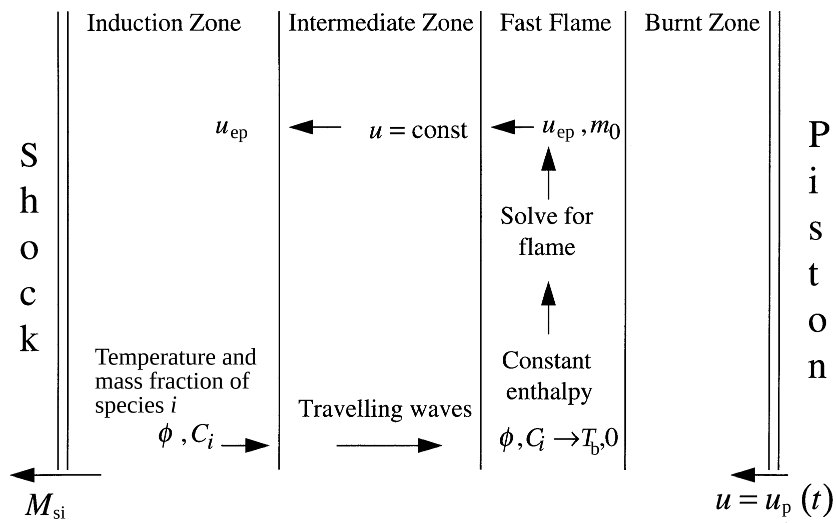


Figure 7. Shock–fast-flame interactions.

transmission to the flame, such that a possible route for flame acceleration via flame–shock interactions exists which does not depend on two-dimensional effects.

### (*c*) *Acoustic feedback and ignition*

Further numerical studies of the interaction of a shock wave with an existing fast flame have been made to investigate whether a different type of ignition event can occur, whereby the thermal singularity occurs on the induction zone side of the intermediate zone connecting this to the fast flame and piston. Normally, the ignition event occurs just ahead of the fast flame itself (see figure 8). If secondary ignition occurs (when the ignition event occurs at the induction zone side of the fast-flame region), then this will lead to a different type of behaviour—that is, separate reaction centres ahead of the first front. Consequently, the possibility of rapid self-sustained acceleration of the front emerges due to a sequence of *forward repeated secondary*

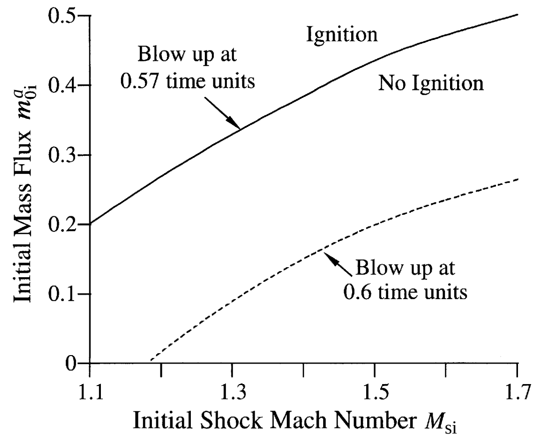


Figure 8. The boundary for ignition (when reduced temperature $\phi = 5$) plotted for activation energy $\theta = 10$. Contours for two blow-up times are shown. The flame is initially propagating with a non-dimensional mass flux $m^a_{0i}$ (non-dimensionalized with respect to the initial density of the cold reactants, and sound speed).

*ignition* events, leading to a detonation. Techniques for path following of the singularity have been developed for the case of initiation with no initial flame present (Parkins 2000; Parkins *et al.* 2000). The present investigations, where there is an initial (fast) flame already present, require the application of similar approaches to show the development into a detonation. If the ignition event can be shown for some initial conditions to take place ahead of the flame, then the nonlinear feedback between the existing fast flame and this process (shock initiation) will have great impact on the speed of development of the subsequent blow-up event(s) leading to a detonation.

## 5. Compressible subsonic deflagrations and large-length-scale ($N \gg M^{-1}$) acoustic interactions

Following on from the last section, it is pertinent to ask where forward travelling shock waves might emanate from, in order to bring about repeated secondary ignition and subsequent acceleration of the combustion front. In addressing this point, there are two possibilities. Firstly, a fast flame with an induction zone ahead will eventually lead inexorably to a thermal singularity due to the acoustic/reactive nature of the Clarke equation governing the induction zone. This is apart from any further disturbance being imposed externally. Nevertheless, there is an important alternative scenario, whereby a combustion front is travelling at fast subsonic speeds, which may not *necessarily* be a fast flame of the Clarke variety. Useful as that model is, there is much experimental evidence (Lee 1986; Scarinci *et al.* 1993) that fast combustion fronts are invariably turbulent in nature. Hence the structure of the fast combustion front is open to a certain extent. Ni & Goel (1995) have explored this matter by investigating the perturbation of the complete system of Euler equations ahead and behind a fast subsonic (compressible) front including the Rankine–Hugoniot jump conditions. With entropy no longer approximately conserved across the front due to the high speed, the perturbation in pressure just before and after is now also not the same, but one can obtain a connection between the amplitude of (large-length-scale ($N \gg M^{-1}$)) acoustic disturbances before and behind the front, which has (as a parameter) the dependence of burning rate on pressure. Thus the whole theory can be applied to fast turbulent, as well as conventional laminar, structures. For fast turbulent flames it is recognized that the mean mass burning rate may itself vary due to small but significant baroclinic terms. In this simple model, one can only input a given sensitivity of mass burning rate with pressure. The one-dimensional assumption where the combustion wave is treated as a 'black box' is simply a useful tool very much in the spirit of the well-established Rankine–Hugoniot theory, which is surprisingly accurate in its predictive power of such measurable quantities as the Chapman–Jouguet detonation velocity, when the actual structure of a detonation is exceedingly non-planar. In the same spirit, we apply the theory to fast subsonic deflagrations (where the structure is not necessarily known) in order to get some estimate as to regions of acoustic resonance. What follows is a brief summary of the main results, with an important derivation of the resonance frequency condition for fast compressible combustion waves using the approach of Ni & Goel (1995).

### (*a*) *One-dimensional acoustic instability of a fast compressible combustion front*

The usual Rankine–Hugoniot relationships (Strehlow 1985 (see pp. 127–138 for a discussion of Rankine–Hugoniot theory)) across a compressible front can be expressed as

$$p_2 = p_1 + \gamma m_0^{a^2}(v_1 + v_2) \qquad \text{(Rayleigh line)}, \tag{5.1}$$

$$p_1 v_1 - p_2 v_2 = \tfrac{1}{2}(1 - \gamma^{-1})(v_1 + v_2)(p_1 - p_2) - Q^* \qquad \text{(Hugoniot curve)}, \tag{5.2}$$

where $v$ is specific volume (inverse density) and $Q^*$ is non-dimensionalized heat release with respect to $c_p' T_{01}'$—that is, using upstream temperature. The continuity of mass flux gives a third relationship,

$$\frac{u_1^a - \hat{\beta}_t}{v_1} = \frac{u_2^a - \hat{\beta}_t}{v_2} \equiv m_0^a \quad \text{(mass continuity)}, \tag{5.3}$$

where all quantities are non-dimensionalized with respect to steady upstream (cold) values, except for velocity $u^a$, which is non-dimensionalized with respect to the initial upstream value of the speed of sound. Consequently, the steady value of mass flux $m_0^a$ in this notation is $M_{01}$, where $M_{01}$ and $M_{02}$ are the initial steady inlet Mach number and outlet Mach number, respectively. The term $\hat{\beta}_t$ is the time derivative of the flame position, which under steady state conditions initially is simply $-M_{01}$.

Eliminating $v_2$ between equations (5.1) and (5.2) yields

$$-p_2(p_1 - p_2) - \tfrac{1}{2}(1 - \gamma^{-1})(p_1 - p_2)^2 = -\gamma m_0^{a^2} Q^* - m_0^{a^2} v_1(p_1 - p_2), \tag{5.4}$$

so that $m_0^a$ can now be considered as a function of $p_1$, $p_2$ and $v_1$. We now consider small perturbations

$$p_1 = p_{s1} + \hat{p}_{u1}, \tag{5.5 a}$$

$$p_2 = p_{s2} + \hat{p}_{u2}, \tag{5.5 b}$$

$$v_1 = v_{s1} + \hat{v}_{u1}, \tag{5.5 c}$$

$$m_0^a = M_{01} + \hat{m}_u^a, \tag{5.5 d}$$

$$u_1^a = u_{s1}^a + \hat{u}_{u1}^a, \tag{5.5 e}$$

$$u_2^a = u_{s2}^a + \hat{u}_{u2}^a, \tag{5.5 f}$$

where it should be noted that $v_2$ is now a derived quantity, since compressibility is significant, and entropy ($s \equiv vp^{\gamma^{-1}}$) is not conserved across the front. Effectively, this route avoids considering the entropy perturbation $(s_2 - s_1)$ directly.

With these small perturbations, the Euler equations in the acoustic zones either side yield the linear acoustic wave equation to leading order, with the connection of velocity and pressure,

$$\frac{\partial^2 \hat{p}_{\mathrm{u1}}}{\partial \hat{x}^2} = \frac{v_{\mathrm{s1}}}{p_{\mathrm{s1}}} \frac{\partial^2 \hat{p}_{\mathrm{u1}}}{\partial \hat{t}^2}, \tag{5.6 a}$$

$$\frac{\partial^2 \hat{p}_{\mathrm{u2}}}{\partial \hat{x}^2} = \frac{v_{\mathrm{s2}}}{p_{\mathrm{s2}}} \frac{\partial^2 \hat{p}_{\mathrm{u2}}}{\partial \hat{t}^2}, \tag{5.6 b}$$

$$\frac{\partial \hat{u}^a_{\mathrm{u1}}}{\partial \hat{t}} = -\frac{1}{\gamma} \frac{\partial \hat{p}_{\mathrm{u1}}}{\partial \hat{x}}, \tag{5.7 a}$$

$$\frac{\partial \hat{u}^a_{\mathrm{u2}}}{\partial \hat{t}} = -\frac{1}{\gamma} \frac{\partial \hat{p}_{\mathrm{u2}}}{\partial \hat{x}}. \tag{5.7 b}$$

Note that in keeping with all the earlier work of the author, $\hat{x}$ and $\hat{t}$ represent the acoustic zone coordinate notation for space and time, and $\hat{x}$ is mass weighted. Approximately $\hat{x} \approx x^a/v_s$, where $x^a$ is the true (unweighted) coordinate in the acoustic zones, so that, as expected to leading order, equations (5.6) become

$$\frac{\partial^2 \hat{p}_{\mathrm{u1}}}{\partial x^{a^2}} \left[ = \frac{1}{v_{\mathrm{s1}} p_{\mathrm{s1}}} \frac{\partial^2 \hat{p}_{\mathrm{u1}}}{\partial \hat{t}^2} \right] = \frac{1}{T_{\mathrm{s1}}} \frac{\partial^2 \hat{p}_{\mathrm{u1}}}{\partial \hat{t}^2}, \tag{5.8 a}$$

$$\frac{\partial^2 \hat{p}_{\mathrm{u2}}}{\partial x^{a^2}} \left[ = \frac{1}{v_{\mathrm{s2}} p_{\mathrm{s2}}} \frac{\partial^2 \hat{p}_{\mathrm{u2}}}{\partial \hat{t}^2} \right] = \frac{1}{T_{\mathrm{s2}}} \frac{\partial^2 \hat{p}_{\mathrm{u2}}}{\partial \hat{t}^2}. \tag{5.8 b}$$

Assuming $v_{\mathrm{s1}} = 1$ and $p_{\mathrm{s1}} = 1$, and noting that $M_{02}^2 = v_{\mathrm{s2}} M_{01}^2 / p_{\mathrm{s2}}$, then linearizing relationships (5.3) and (5.4) yields

$$(v_{\mathrm{s2}} - 1)\hat{m}^a_u = -(\hat{u}^a_{\mathrm{u2}} - \hat{u}^a_{\mathrm{u1}}) - \frac{1}{\gamma M_{01}}(\hat{p}_{\mathrm{u2}} - \hat{p}_{\mathrm{u1}}), \tag{5.9}$$

$$\hat{p}_{\mathrm{u1}} \left[ 1 - \frac{\gamma M_{01}^2}{p_{\mathrm{s2}}}(1 - v_{\mathrm{s2}}) - M_{02}^2 + \frac{2M_{01}^2}{\gamma} - \frac{2M_{01}}{p_{\mathrm{s2}}} \frac{\hat{m}^a_u \gamma Q^*}{\hat{p}_{\mathrm{u1}}} \right] = \hat{p}_{\mathrm{u2}}[1 - M_{02}^2]. \tag{5.10}$$

We now assume harmonic oscillations of the form

$$\hat{p}_{\mathrm{u1}} = p_{\mathrm{u1}} \mathrm{e}^{\omega \hat{t}}, \tag{5.11 a}$$

$$\hat{p}_{\mathrm{u2}} = p_{\mathrm{u2}} \mathrm{e}^{\omega \hat{t}}, \tag{5.11 b}$$

$$\hat{v}_{\mathrm{u1}} = v_{\mathrm{u1}} \mathrm{e}^{\omega \hat{t}}, \tag{5.11 c}$$

$$\hat{u}^a_{\mathrm{u1}} = u^a_{\mathrm{u1}} \mathrm{e}^{\omega \hat{t}}, \tag{5.11 d}$$

$$\hat{u}^a_{\mathrm{u2}} = u^a_{\mathrm{u2}} \mathrm{e}^{\omega \hat{t}}, \tag{5.11 e}$$

$$\hat{m}^a_u = m^a_u \mathrm{e}^{\omega \hat{t}}, \tag{5.11 f}$$

where $\omega \equiv \omega_r + i\omega_i$ is the complex frequency with real ($\omega_r$ is the growth rate) and imaginary ($\omega_i$ is the radial frequency) parts. Noting from equations (5.7) that

$$u_{u1}^a = -\frac{1}{\gamma\omega}\frac{d\hat{p}_{u1}}{d\hat{x}}, \tag{5.12 a}$$

$$u_{u2}^a = -\frac{1}{\gamma\omega}\frac{d\hat{p}_{u2}}{d\hat{x}}, \tag{5.12 b}$$

the connecting relationships (5.9) and (5.10) become

$$p_{u1}\left[\frac{1}{\gamma} - \frac{M_{01}m_u^a(v_{s2}-1)}{p_{u1}}\right] = \frac{p_{u2}}{\gamma} - \frac{M_{01}}{\gamma}\left[\frac{1}{\omega}\frac{dp_{u2}}{d\hat{x}}\bigg|_0 - \frac{1}{\omega}\frac{dp_{u1}}{d\hat{x}}\bigg|_0\right], \tag{5.13}$$

$$p_{u1}\left[1 + \frac{\gamma M_{01}^2}{p_{s2}}(v_{s2}-1) - M_{02}^2 + \frac{2M_{01}^2}{\gamma} - \frac{2M_{01}}{p_{s2}}\frac{\hat{m}_u^a\gamma Q^*}{p_{u1}}\right] = p_{u2}[1 - M_{02}^2]. \tag{5.14}$$

The harmonic solution to equations (5.6) using (5.11) is of the form

$$p_{u1} = A_1 e^{\omega\hat{x}} + B_1 e^{-\omega\hat{x}}, \tag{5.15 a}$$

$$p_{u2} = A_2 e^{\omega\hat{x}\sqrt{v_{s2}/p_{s2}}} + B_2 e^{-\omega\hat{x}\sqrt{v_{s2}/p_{s2}}}, \tag{5.15 b}$$

so that, using $M_{02}^2 = v_{s2}M_{01}^2/p_{s2}$, equations (5.13) and (5.14) become

$$(A_1 + B_1)\left[1 - \frac{M_{01}p_{s2}(v_{s2}-1)\aleph}{Q^*}\right] = (A_2 + B_2) - [M_{02}(A_2 - B_2) - M_{01}(A_1 - B_1)], \tag{5.16}$$

$$(A_1 + B_1)\left[1 - M_{02}^2 - 2M_{01}\aleph + \frac{2M_{01}^2}{\gamma} + \frac{\gamma M_{01}^2}{p_{s2}}(v_{s2}-1)\right] = (A_2 + B_2)[1 - M_{02}^2], \tag{5.17}$$

where the important parameter defining the sensitivity of mass burning rate to pressure change is

$$\aleph \equiv \frac{m_u^a}{p_{s2}p_{u1}}\gamma Q^*. \tag{5.18}$$

With some manipulation, to leading order (that is, single powers in $M_{01}$ and $M_{02}$), and noting that

$$\frac{Q^*}{(v_{s2}-1)} = 1 - O(M_{01}^2) \tag{5.19 a}$$

and

$$\frac{p_{s2}(v_{s2}-1)}{Q^*} = 1 + O(M_{01}^2), \tag{5.19 b}$$

with the reflection coefficients defined as follows:

| | | | | |
|---|---|---|---|---|
| case (a) | $B_1 = 0$ | (input from the hot side), | $K \equiv B_2/A_2$, | (5.20 *a*) |
| case (b) | $A_2 = 0$ | (input from the cold side), | $L \equiv A_1/B_1$, | (5.20 *b*) |

it can be shown (Ni & Goel 1995) that for the absolute value of the reflection coefficients in cases (a) and (b) above to be greater than one (and consequently to have acoustic growth), the mass burning rate sensitivity to pressure must lie in the following range:

$$\text{case (a)} \qquad K > 1: \quad 1 < \aleph < 1 + \frac{M_{02}}{M_{01}}, \tag{5.21 a}$$

$$\text{case (a)} \qquad K < -1: \quad \aleph > 1 + \frac{M_{02}}{M_{01}}, \tag{5.21 b}$$

$$\text{case (b)} \qquad L > 1: \quad \frac{M_{02}}{M_{01}} < \aleph < 1 + \frac{M_{02}}{M_{01}}, \tag{5.22 a}$$

$$\text{case (b)} \qquad L < -1: \quad \aleph > 1 + \frac{M_{02}}{M_{01}}. \tag{5.22 b}$$

Ni & Goel in fact use the parameter $M_{01}\aleph/M_{02}$. The above $\aleph$ definition in equation (5.18) is used because it can be shown that, to leading order, $\aleph$ is equivalent to the mass flux derivative with pressure times heat release. Thus

$$\aleph \equiv \frac{m_u^a}{p_{s2}p_{u1}}\gamma Q^* \approx \gamma \left[\frac{\partial m_0^a}{\partial p_1}\right]\Bigg|_{s_1}(v_{s2}-1) \approx \gamma \left[\frac{\partial m_0^a}{\partial p_1}\right]\Bigg|_{s_1} Q^*. \tag{5.23}$$

In general, for laminar low-speed flames, the value of $\aleph$ is too small, but as pointed out by Ni & Goel (1995), a fast turbulent flame will quite readily meet the criteria (5.21) and (5.22) for resonance.

Thus the above criteria are important for acoustic growth. Take for example case (a) with $K > 1$. If there is a wall on the hot side of the combustion front, then for an initial perturbation approaching from the hot side with $1 < \aleph < 1 + M_{02}/M_{01}$, the reflection back to the wall is stronger, whereupon as it interacts with the combustion front a second time, there is further growth. A similar situation can arise with case (b) with $L > 1$ and $M_{02}/M_{01} < \aleph < 1 + M_{02}/M_{01}$, but this time a wall on the cold side. Indeed, this case will be very common, since it represents a combustion front entering a combustible mixture, closed at the cold end.

### (*b*) *Resonance frequency condition for fast compressible combustion waves*

Returning to results (5.16) and (5.17), and adding tube conditions at $x^a = -\ell_1$ and $x^a = \ell_2$ ($\hat{x} = \ell_2/v_{s2}$), one can derive a tube frequency condition for resonance. For the open tube case ($p_{u1}(\hat{x} = -\ell_1) = 0$ and $p_{u2}(\hat{x} = \ell_2/v_{s2}) = 0$), the frequency condition that emerges to leading order is given by

$$\frac{\cosh(\omega\ell_1)}{\sinh(\omega\ell_1)} + \frac{M_{02}}{M_{01}}\frac{\cosh(\omega\ell_2/\sqrt{T_{s_2}})}{\sinh(\omega\ell_2/\sqrt{T_{s_2}})} = \aleph. \tag{5.24}$$

Conditions like these have to be solved for real ($\omega_r$) and imaginary ($\omega_i$) parts of complex frequency $\omega$ to ascertain the regions of $\aleph$–$\ell_1$–$\ell_2$ space where resonance ($\omega_r > 0$) is likely to occur.

Similar frequency conditions can be found for other types of far boundary conditions. The growth of amplitude of the acoustic waves will, of course, in practice be limited by nonlinear dissipative (viscous) terms, and the occurrence of resonance amplifying such long-wavelength signals will further create a greater likelihood of an ignition event ahead of the flame front due to the effects described in § 4 *c*.

## 6. Conclusions

An overview has been presented of the important one-dimensional pressure interactions that can occur with combustion fronts. The three main results that should be noted are as follows.

(1) There is an important high-frequency natural resonance for premixed flames with low-amplitude acoustics, which is apart from any external geometrical (organ pipe) interference. Such a pure tone resonance is awaiting experimental verification, and should lie somewhere in the region of 2–3 kHz for typical hydrocarbon mixtures.

(2) An important mechanism for flame acceleration is the possibility of forward repeated secondary ignition, where small acoustic disturbances are amplified in the induction zone of a fast (local) convection–reaction driven combustion front. If such amplification can be confirmed for a piston driven into a combustible mixture (with a fast flame already in existence), then this will lead to a temperature singularity away from the main combustion front and can be a possible mechanism for transition to a detonation.

(3) As demonstrated first by Ni & Goel (1995), long-wavelength acoustic waves can still amplify away from the reactive zone, if the whole combustion front is travelling at velocities that generate sufficient compressibility. The structure of the front in this case is not necessarily specified as being of a particular kind, and the theory can therefore readily be applied to turbulent flames for which this type of effect is likely to occur.

## References

Batley, G. A., McIntosh, A. C. & Brindley, J. 1993 The time evolution of interactions between short length scale pressure disturbances and premixed flames. *Comb. Sci. Technol.* **92**, 367–388.

Blythe, P. & Crighton, D. G. 1989 Shock-generated ignition—the induction zone. *Proc. R. Soc. Lond.* A **426**, 189–209.

Booty, M. R. 1994 Modulation of fast deflagrations. Poster at 25th Int. Symp. on Combustion, Irvine, CA, USA.

Booty, M. R. 1996 Time dependent premixed deflagrations. AIAA Paper 96-0910. In *Proc. 24th Aerospace Sciences Meeting, Reno, NV, USA.*

Chue, R. S., Clarke, J. F. & Lee, J. H. 1993 Chapman–Jouguet deflagrations. *Proc. R. Soc. Lond.* A **441**, 607–623.

Clarke, J. F. 1983 On changes in the structure of steady plane flames as their speed increases. *Comb. Flame* **50**, 125–138.

Clarke, J. F. & Cant, R. S. 1984 Non-steady gas dynamic effects in the induction domain behind a strong shock wave. *Prog. Astronaut. Aeronaut.* **95**, 142–163.

Dold, J. W., Kapila, A. K. & Short, M. 1991 Theoretical description of direct initiation of detonation for one-step chemistry. In *Dynamic structure of detonation in gaseous and dispersed media* (ed. A. A. Borisov), pp. 109–141. Dordrecht: Kluwer Academic.

Dold, J. W., Short, M., Clarke, J. F. & Nikiforakis, N. 1995 Accumulating sequence of ignitions from a propagating pulse. *Comb. Flame* **100**, 465–473.

Incropera, F. P. & DeWitt, D. P. 1996 *Fundamentals of heat and mass transfer*, 4th edn. Wiley.

Johnson, R. G. & McIntosh, A. C. 1995 On the phenomenon of local flow reversal in premixed flames due to a large pressure pulse. *Comb. Sci. Technol.* **104**, 287–294.

Johnson, R. G., McIntosh, A. C. & Brindley, J. 1995 Extinction of premixed flames by pressure drops. *Comb. Flame* **102**, 493–500.

Johnson, R. G., McIntosh, A. C. & Brindley, J. 1996*a* Evolution of a fast flame behind an induction zone. *Shock Waves* **6**, 225–231.

Johnson, R. G., McIntosh, A. C., Brindley, J., Booty, M. R. & Short, M. 1996*b* Shock-wave interaction with a fast convection–reaction driven flame. In *Proc. Int. 26th Symp. on Combustion*, pp. 891–898.

Kassoy, D. R. & Clarke, J. F. 1985 The structure of a steady high-speed deflagration with a finite origin. *J. Fluid Mech.* **150**, 253–280.

Ledder, G. & Kapila, A. K. 1991 The response of premixed flames to pressure perturbations. *Comb. Sci. Technol.* **75**, 21–44.

Lee, J. H. S. 1986 On the transition from deflagration to detonation. In *Dynamics of explosions: AIAA progress in astronautics and aeronautics* (ed. J. R. Bowen *et al.*), vol. 106, pp. 3–11. New York: AIAA.

McIntosh, A. C. 1991 Pressure disturbances of different length scale interacting with conventional flames. *Comb. Sci. Technol.* **75**, 287–309.

McIntosh, A. C. 1993 The linearised response of the mass burning rate of a premixed flame to rapid pressure changes. *Comb. Sci. Technol.* **91**, 329–346.

McIntosh, A. C. 1995*a* Pressure-driven disturbances in fluid–dynamic interactions with flames. In *Proc. Joint US (NSF)/Japan (JSPS) workshop on Mathematical Modelling in Combustion, Kauai, Hawaii, 25–29 July 1994.* In *Modelling in combustion science* (ed. J. Buckmaster & T. Takeno). Springer Lecture Notes in Physics, pp. 176–192.

McIntosh, A. C. 1995*b* The influence of pressure waves on the initial development of an explosion kernel. *AIAA J.* **33**, 1651–1658.

Ni, A. L. & Goel, B. 1995 Stability of a deflagration front in compressible media. *Z. Angew. Math. Phys. (ZAMP)* **46**, 328–338.

Parkins, C. J. 2000 Shock generated ignition: Newtonian asymptotics for the induction domain between a contact surface and a shock. *SIAM J. Appl. Math.* (In the press.)

Parkins, C. J., Blythe, P. & Crighton, D. G. 2000 Hot spot ignition: the Newtonian limit. *Proc. R. Soc. Lond.* A. (In the press.)

Scarinci, T., Lee, J. H., Thomas, G. O., Bambrey, R. & Edwards, D. H. 1993 Amplification of a pressure wave by its passage through a flame front. *AIAA Prog. Astron. Aeronaut.* **153**, 3–24.

Singh, G. & Clarke, J. F. 1992 Transient phenomena in the initiation of a mechanically driven plane detonation. *Proc. R. Soc. Lond.* A **438**, 23–46.

Short, M. & Dold, J. W. 1993 Compressibility corrections to Zeldovich's spontaneous flame and the onset of an explosion in a non-uniformly preheated medium. *Prog. Astron. Aeronaut.* **154**, 59–74.

Strehlow, R. A. 1985 *Combustion fundamentals*, 2nd edn. McGraw-Hill.